Lecture 14 - Oct 28

Bridge Controller

Justifying the OR_L Inference Rule
Interpreting Unprovable Sequents

Announcements/Reminders

- Today's class: notes template posted
- ProgTest and WrittenTest1 results released
- Tomorrow's lab sessions (1:30 to 3:30):
  Shangru to go over parts of your ProgTest
- Lab3 released

P : Peano Numbers theorems

Example Inference Rules

axiom rules (base cases of a sequent proof)

(recursive cases of sequent proofs)

P1

$$\frac{}{\vdash 0 \in \mathbb{N}}$$

P2

$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}$$

INC

n m

$$\frac{}{\boxed{n < m \vdash n+1 \leq m}}$$

e.g. $n=2$, $m=3$, $\vdash n+1 \leq m$

P2'

0 1 n

$$\frac{}{0 < n \vdash n-1 \in \mathbb{N}}$$

DEC

① $n=2$ $m=3$ $n-1=1$ ② $n=m=3$

$$\frac{}{\boxed{n \leq m} \vdash n-1 < m}$$

① $n<m$ ② $n=m$

P3

$$\frac{}{n \in \mathbb{N} \vdash 0 \leq n}$$

OR_L

$$\frac{H, P \vdash R \qquad H, Q \vdash R}{H, P \vee Q \vdash R}$$

OR_R1

to the R of $\vdash$

$$\frac{H \vdash P}{H \vdash P \vee Q}$$

1

OR_R2

$$\frac{H \vdash Q}{H \vdash P \vee Q}$$

2

MON

$$\frac{H1 \vdash G}{H1, H2 \vdash G}$$

$$\frac{H \vdash P}{H \vdash P \lor Q} \quad \textbf{OR\_R1}$$

R1

$H \vdash P \lor Q$ OR_R2 $H \vdash Q$

$$\frac{H \vdash Q}{H \vdash P \lor Q} \quad \textbf{OR\_R2}$$

R2

$H \vdash$ ✓ $P \lor Q$ OR_R1 $H \vdash P$

Justifying Inference Rule: OR_L

$$\frac{\cancel{H}, P \vdash R \quad \wedge \quad \cancel{H}, Q \vdash R}{\cancel{H}, \boxed{P \vee Q \vdash R}} \quad \textbf{OR\_L}$$

$$\big((P \Rightarrow R) \wedge (Q \Rightarrow R) \Rightarrow P \vee Q \Rightarrow R\big) \equiv \text{True.}$$

$(P \Rightarrow R) \wedge (Q \Rightarrow R)$

$\equiv \{$ def. of imp: $x \Rightarrow y \equiv \neg x \vee y \}$

$(\neg P \vee R) \wedge (\neg Q \vee R)$

$\equiv <$ distributivity of $\vee$ over $\wedge$: $x \vee (y \wedge z) \equiv (x \vee y) \wedge (x \vee z) \}$

$R \vee (\neg P \wedge \neg Q)$

$\equiv \{$ de morgan: $\neg(x \vee y) \equiv \neg x \wedge \neg y \}$

y $\boxed{\boxed{R} \vee \neg\boxed{(P \vee Q)}}$ x $\equiv \{$ def. of imp. $\}$ $P \vee Q \Rightarrow R$

Discharging POs of original m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n + 1 \in \mathbb{N}$

MON

$n \in \mathbb{N} \vdash n+1 \in \mathbb{N}$

P2

P2 ✗ not valid ∵ the consequent of P2 has exactly one hypothesis

ML_in/inv0_1/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n - 1 \in \mathbb{N}$

MON

$n \in \mathbb{N} \vdash n-1 \in \mathbb{N}$

$n > 0$

unprovable ∵ $n = 0$ then $n-1 \notin \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n + 1 \leq d$

MON

$n \leq d \vdash n < d$
$n+1 \leq d$

not provable $n = d$

extra hypothesis needed to prove.

ML_in/inv0_2/INV

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \leq d$
$\vdash$
$n - 1 \boxed{\leq} d$

ARI

arithmetic ($a \leq b \equiv a < b \vee a = b$)

$d \in \mathbb{N}$, $n \in \mathbb{N}$, $\boxed{n \leq d} \vdash n-1 < d \vee n-1 = d$

MON

$n \leq d \vdash n-1 < d \vee n-1 = d$

OR_R1

$n \leq d \vdash n-1 < d$

DEC

$$\frac{H \vdash P}{H \vdash P \vee Q}\ \textbf{OR\_R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G}\ \textbf{MON}$$

$$\frac{}{n \leq m \vdash n - 1 < m}\ \textbf{DEC}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}}\ \textbf{P2}$$

ML_in/inv0_1/INV.

$n' = n-1$

$d \in \mathbb{N}$
$n \in \mathbb{N}$
$n \le d$
$\vdash$ True
$n - 1 \in \mathbb{N}$

given that this sequent is unprovable; we may want to add some additional hypothesis to help.

$n > 0$

where should this hypothesis go in the model?

constants

$n > 0$

variables

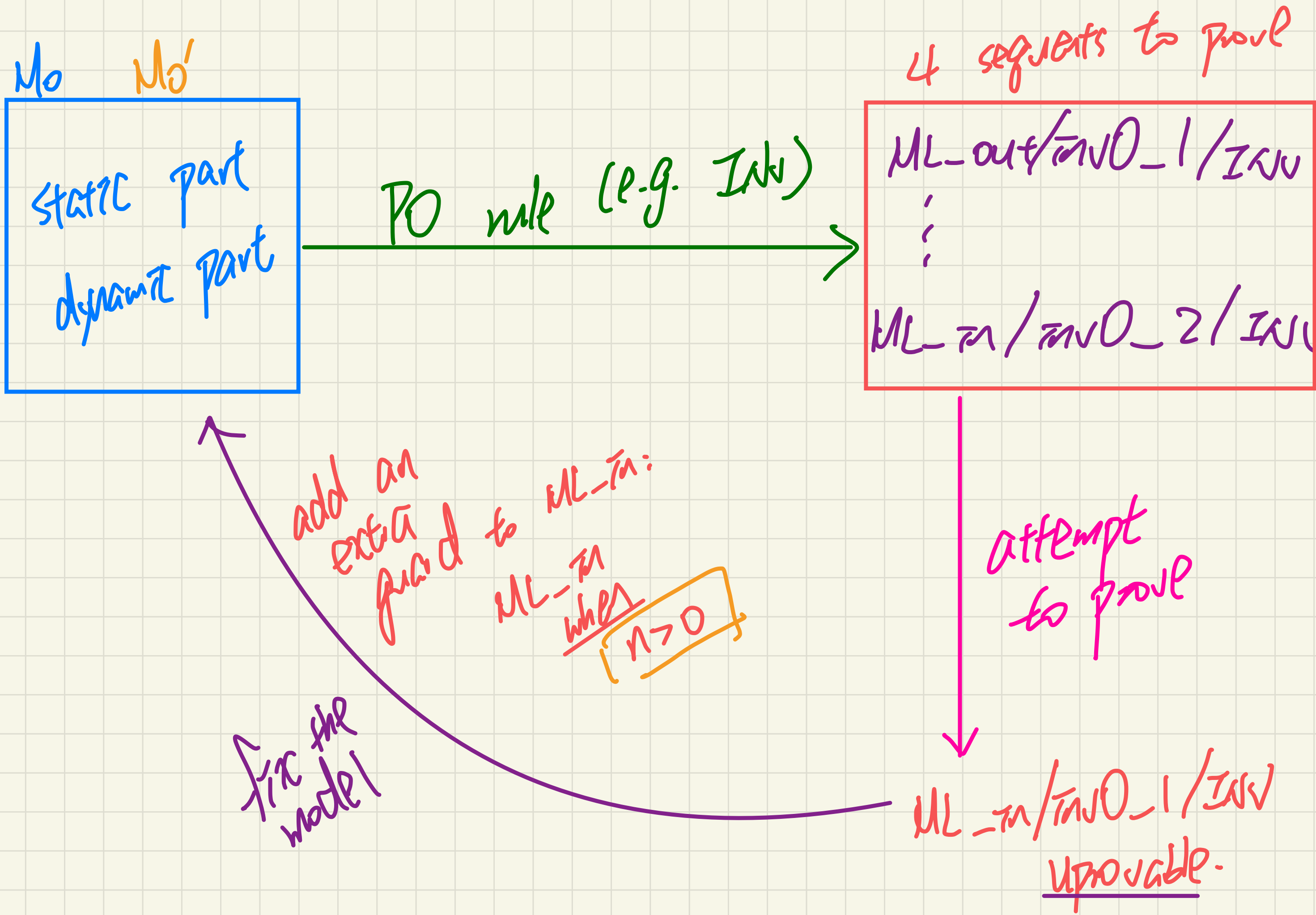
① ②

$A(c)$
$I(c, v)$ $n>0$
$G(c, v)$ ML_in
$\vdash$
$I_i(c, E(c, v))$

this is the only suitable place to add the extra hypothesis.

ML_in when $n > 0$ begin ... end

extra guard.


M0
M0'
static part
dynamic part
PO rule (e.g. INV)
4 sequents to prove
ML_out/inv0_1/INV
⋮
ML_in/inv0_2/INV
attempt to prove
ML_in/inv0_1/INV
unprovable.
add an extra guard to ML_in:
ML_in when n>0
fix the model